EQUIVALENCE SCALES

(A. Vernizzi 2015)

Equivalence scales are particular economic indexes.

They are used to compare *the value* which a same monetary amount can have when family needs are different.

Equivalence scales reproduce the income levels which families with different needs should have in order to enjoy the same welfare level.

(Different needs: number of components, age, health conditions, characteristics of the surrounding environment, etc.)

Equivalence scales are generally estimated using three different approaches:

(a) by considering the expense for food and other basic goods strictly necessary to survive, for families having different characteristics;

(b) by observing (or describing) consumption structures (and consequently their cost) of families with different characteristics;

(c) by the empirical analysis of the consumption behaviour of families, observed as consumption units.

THE ENGEL LAWS (1895)

The ratio $\dfrac{Food\ expenditure}{Total\ consumption\ expenditure}$

or the ratio $\dfrac{Food\ expenditure}{Income}$

1) is an inverse function of total consumption (of income), ceteris paribus;

2) is a direct function of the family components, ceteris paribus.

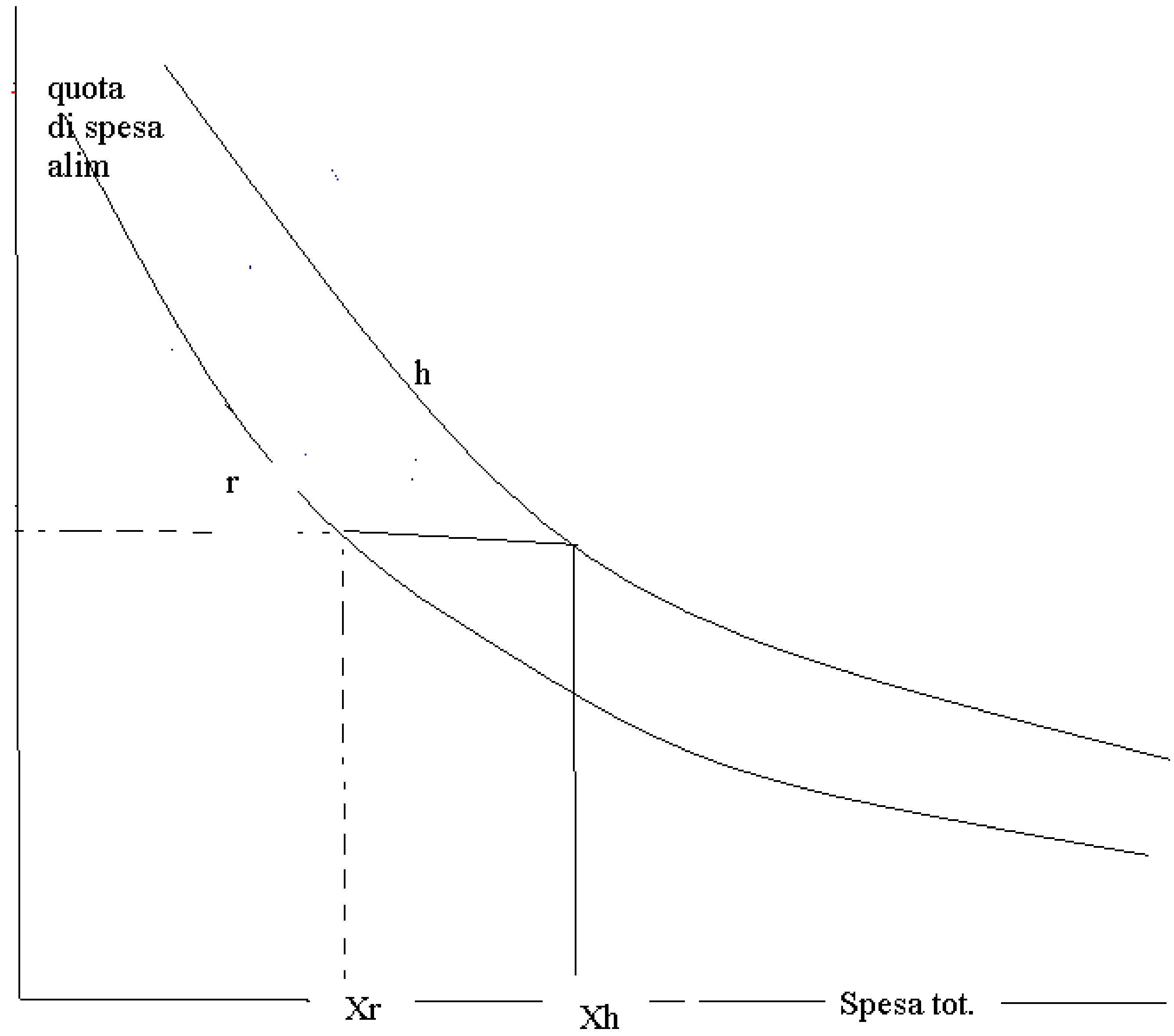

Fig. 1

Fig. 1, *y*-axis: the expenditure share devoted to food;
Fig. 1, *x*-axis: total expenditure (or income)

On the basis of the Engel *laws* one can conclude that two families enjoy the same welfare level, when they present the same expenditure (income) share spent on food
:
X_h/X_r indicates how many times family *h*'s total consumption (income) should be greater (lower) than that of family *r*, in order that family *h*'s welfare is the same as that of family *r*.

In the pursue, for the sake of simplicity, we consider just scales which are function of family components:

$$ {}_rS_h(u_0) = {}_rS_h = f(n_h, n_r) $$

The starting point is the following equation (Working-Leser, 1943, 1963):

$$\log C_{a,h} = \alpha + \beta \log X_h + \eta \log n_h \qquad (1)$$

$C_{a,h}$ is the monetary expenditure for food of family *h*;
X_h is the total monetary expenditure of family *h*;
n_h is the number of persons in family *h*.

The Engels laws deal with food expenditure *shares*, not food expenditures. So equation (1) has to be manipulated:

$$\log C_{a,h} - \log X_h = \alpha + \beta \log X_h - \log X_h + \eta \log n_h$$

$$\log \frac{C_{a,h}}{X_h} = \alpha + (\beta - 1) \log X_h + \eta \log n_h$$

$$\log W_{a,h} = \alpha + (\beta - 1) \log X_h + \eta \log n_h$$

By assuming that family h and family r, have the same welfare level if $W_{a,h} = W_{a,r}$, or, which is the same, being $log(\cdot)$ a monotonic transformation, $\log W_{a,h} = \log W_{a,r}$, we have

$$\log W_{a,h} = \log W_{a,r} \quad \rightarrow$$

$$\alpha + (\beta - 1)\log X_h + \eta \log n_h = \alpha + (\beta - 1)\log X_r + \eta \log n_r$$

$$(\beta - 1)\log X_h - (\beta - 1)\log X_r = -\eta \log n_h + \eta \log n_r$$

$$\log \frac{X_h}{X_r} = -\frac{\eta}{(\beta - 1)} \log \frac{n_h}{n_r} \qquad \log \frac{X_h}{X_r} = \frac{\eta}{(1 - \beta)} \log \frac{n_h}{n_r}$$

$$\frac{X_h}{X_r} = \left(\frac{n_h}{n_r} \right)^{\frac{\eta}{(1-\beta)}} = {}_r S_h$$

Remark:

$$ {}_5S_2 = \frac{1}{{}_2S_5}; \qquad {}_2S_5 = \frac{{}_1S_5}{{}_1S_2}; \qquad {}_1S_5 = {}_1S_2 \cdot {}_2S_5 $$

One can estimate directly the equation

$$ \log W_{a,h} = \alpha + \gamma \log X_h + \eta \log n_h \qquad (2) $$

From which, by setting $\log W_{a,h} = \log W_{a,r}$, one yields

$$ \frac{X_h}{X_r} = \left(\frac{n_h}{n_r}\right)^{-\frac{\eta}{\gamma}} = {}_rS_h $$

With $\gamma = (\beta - 1)$, α and β are the same as in (1).

Alternatively one can start from a different specification as:

$$W_{a,h} = \tau + \pi \log X_h + \varphi \log n_h \qquad (3)$$

From (3), by setting $W_{a,h} = W_{a,r}$, one derives

$$\frac{X_h}{X_r} = \left(\frac{n_h}{n_r}\right)^{-\frac{\varphi}{\pi}} = {}_rS_h$$

Remark: in general $\tau \neq \alpha$, $\pi \neq \gamma$ (i.e. $\pi \neq \beta - 1$), $\varphi \neq \eta$, and no direct link exists between the coefficient of equations (1) and (2) , on one hand, and those of equation (3) on the other hand.

A SIMPLE EXAMPLE

Tab. 1

1 ***Food expenditure*** *(euro x month)*	2 ***Total consumption expenditure*** *(euro x month)*	3 ***Family components***	4 ***Food/Total Expenditure***	5 *Ln*(***Total expenditure***)	6 *Ln*(***Components***)
235.97	943.88	1	0.25	6.85	0.0000
292.08	1043.15	2	0.28	6.95	0.6931
311.27	1152.86	2	0.27	7.05	0.6931
369.49	1274.11	3	0.29	7.15	1.0986
394.27	1408.11	3	0.28	7.25	1.0986
435.74	1556.20	4	0.28	7.35	1.3863
498.76	1719.86	5	0.29	7.45	1.6094

7 *Ln*(***Food expenditure***)	*Ln*(***Food***/***Total Expenditure***)
5.46	-1.3863
5.68	-1.2730
5.74	-1.3093
5.91	-1.2379
5.98	-1.2730
6.08	-1.2730
6.21	-1.2379

A) OLS estimation

$$\log C_{a,h} = \alpha + \beta \log X_h + \eta \log n_h$$

$$\log\left(\hat{C}_{a,h}\right) = 1.037 + 0.646 \cdot \log\left(X_h\right) + 0.223 \cdot \log\left(n_h\right).$$

from the above estimated function:

$$\log\left(\frac{\hat{C}_{a,h}}{X_h}\right) = 1.037 + \left(0.646 - 1\right) \cdot \log\left(X_h\right) + 0.223 \cdot \log\left(n_h\right)$$

$$\log\left(\hat{W}_{a,h}\right) = 1.037 - 0.354 \cdot \log\left(X_h\right) + 0.223 \cdot \log\left(n_h\right)$$

Let’s derive the equivalence scale, ${}_2S_3$, for family *h*, 3 components, with respect to family *r*, 1 component:

$$1.037 - 0.354 \cdot \log\left(X_3\right) + 0.223 \cdot \log\left(3\right) =$$
$$= 1.037 - 0.354 \cdot \log\left(X_1\right) + 0.223 \cdot \log\left(1\right)$$

$$\log\left(\frac{X_3}{X_1}\right) = \frac{0.223}{0.354}\log\left(\frac{3}{1}\right) \rightarrow \left(\frac{X_3}{X_1}\right) = {}_1S_3 = \left(\frac{3}{1}\right)^{\frac{0.223}{0.354}} = 3^{0.63}$$

B) If we apply ordinary least squares to the same data used in estimating equation (1), reported above in **A**), we obtain

$$\log\left(\hat{W}_{a,h}\right)=1.037-0.354\cdot\log\left(X_h\right)+0.223\cdot\log\left(n_h\right)$$

As we can see we yield the same estimates for the intercept and the coefficient associated to $\log\left(n_h\right)$;

For what concerns the coefficients associated to $\log\left(X_h\right)$, we have:

$-0.354=0.646-1$

By applying the ordinary least squares to equation (3), with the same data, we would obtain:

C)

$$\hat{W}_{a,h} = 0.896 - 0.094 \cdot \log\left(X_h\right) + 0.060 \cdot \log\left(n_h\right)$$

${}_1S_3$ is obtained by the following passages

$$0.896 - 0.094 \cdot \log\left(X_3\right) + 0.060 \cdot \log\left(3\right) =$$
$$= 0.896 - 0.094 \cdot \log\left(X_1\right) + 0.060 \cdot \log\left(1\right)$$

$$\log\left(\frac{X_3}{X_1}\right) = \frac{0,060}{0,094}\log\left(\frac{3}{1}\right) \rightarrow \frac{X_3}{X_1} = {}_1S_3 = \left(\frac{3}{1}\right)^{\frac{0.060}{0.094}} = 3^{0.64}$$

	S	C	P+1ch	P+2 ch	P+3ch
$\left(\frac{n_h}{1}\right)^{0.63}$	$1^{0.63}=1$	$2^{0.63}=1.55$	$3^{0.63}=2.00$	$4^{0.63}=2.40$	$5^{0.63}=2.76$
French Quotient	1	2	2.5	3	4

Alfred Sauvy (Villeneuve-de-la-Raho, October 31st 1898, – Paris, October 30th 1990), the father of the *French quotient*, surely had in mind economic equivalence scales, but not only them, when he designed his *political* equivalence scale.