THE ATKINSON INDEX

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$$A_e = 1 - \frac{1}{\bar{x}}\left[\frac{1}{N}\sum_{i=1}^{N} x_i^{1-e}\right]^{\frac{1}{1-e}} = 1 - \frac{\bar{x}_{1-e}}{\bar{x}}$$

$\bar{x}$ is the average income of N units,

x_i is the income related to unit i (i=1,2,..., N).

$e>0$

$0 \leq A_e \leq 1$

Consider K units, each of them with a particular weight n_k;

$$\sum_{k=1}^{K} n_k = N$$

When considering weights associated to each unit, the index becomes:

$$A_e = 1 - \frac{1}{\bar{x}}\left[\frac{1}{N}\sum_{k=1}^{K} x_k^{1-e} n_k\right]^{\frac{1}{1-e}} = 1 - \frac{\bar{x}_{1-e}}{\bar{x}}$$

$$\bar{x}_{1-e} = \left[\frac{1}{N}\sum_{k=1}^{K} x_k^{1-e} n_k\right]$$

$e = 0$

$$A_0 = 1 - \frac{1}{\bar{x}}\left[\frac{1}{N}\sum_{k=1}^{K} x_k n_k\right] = 1 - \frac{\bar{x}_1}{\bar{x}} = 1 - \frac{\bar{x}}{\bar{x}} = 0$$

$e = 1$

$$A_1 = 1 - \frac{1}{\bar{x}}\lim_{e\to 1}\left\{\left[\frac{1}{N}\sum_{k=1}^{K} x_k^{1-e} n_k\right]^{\frac{1}{1-e}}\right\} = 1 - \frac{1}{\bar{x}}\left[\prod_{k=1}^{K} x_k^{n_k}\right]^{\frac{1}{N}} = 1 - \frac{\bar{x}_0}{\bar{x}}$$

$e = 2$

$$A_2 = 1 - \frac{1}{\bar{x}}\left[\frac{1}{N}\sum_{k=1}^{K} x_k^{-1} n_k\right]^{-1} = 1 - \frac{1}{\bar{x}} \cdot \frac{N}{\sum_{k=1}^{K}\frac{n_k}{x_k}} = 1 - \frac{\bar{x}_{-1}}{\bar{x}}$$

Let's consider two units, *a* and *b*:

a has income zero, *b* has income $x_b \neq 0$; adopt $e = 2$.

$$\bar{x} = \frac{x_a + x_b}{2} = \frac{0 + x_b}{2} = \frac{x_b}{2}$$

$$\bar{x}_{-1} = \lim_{x_a \to 0} \left[\frac{1}{2} \left(x_a^{-1} + x_b^{-1} \right) \right]^{-1} = \lim_{x_a \to 0} \frac{2}{\frac{1}{x_a} + \frac{1}{x_b}} = 0$$

$$\left(\lim_{x_a \to 0} \frac{1}{x_a} = \infty \right)$$

$$A_2 = 1 - \frac{0}{\bar{x}} = 1$$

(i) Due to power mean properties, when (1-*e*)<1, that is to say when *e*>0,

$\longrightarrow \bar{x}_{1-e} < \bar{x}$.

$\bar{x}_{1-e} = \bar{x}$ only when all incomes are equal.

(ii) $\bar{x}_{1-e}$ is an inverse function of *e*.

REMARK

e expresses the subjective evaluation of inequality.

$e=0$ means indifference towards inequality.

The higher ***e***, the greater is subjective aversion against inequality.

$\bar{x}_{1-e}$ can be interpreted as the egalitarian income level which would be acceptable, even if lower of the actual average level $\bar{x}$, provided $\bar{x}_{1-e}$ is the same income level for every units.

E.g., suppose that for a given $e > 0$, $\bar{x}_{1-e}$ is 70% of $\bar{x}$, it means that the politician would lower 30% the population income, provided that a complete egalitarian income distribution is yielded.

Some simple examples.

$$[x_1=2; \qquad x_2=50] \qquad \bar{x}=26$$

$e=0$:

$$A_0 = 1 - \frac{26}{26} = 0$$

$e=1$:

$$\bar{x}_0 = \sqrt{2 \cdot 50} \qquad\qquad A_1 = 1 - \frac{10}{26} = 0,6154$$

<u>*e*=2</u>:

$$\bar{x}_{-1}=\frac{2}{\frac{1}{2}+\frac{1}{50}}=3{,}84615 \qquad A_2=1-\frac{3{,}84615}{26}=0{,}8521$$

<u>*e*=3</u>:

$$\bar{x}_{-2}=\left\{\frac{1}{2}\left[(2)^{-2}+(50)^{-2}\right]\right\}^{-\frac{1}{2}}=2{,}826167; \qquad A_3=1-\frac{2{,}826167}{26}=0.8913$$

Remark on weights n_i

The weights n_i can represent the families components, or the number of equivalent components, calculated by applying a proper equivalence scale.

Consider the example:

Family	*A*	*B*	*C*
Family income	10	12	16
Components n_h	2	3	4
Per-capita income	5	4	4
Equivalent components $(n_h)^{0.65}$	1.569	2.042	2.462
Equivalent income	6.3735	5.8766	6.4988

-If we do not take into account the family components:

x_i: 10 12 16

n_i: 1 1 1 N=3

$\bar{x}$=(10+12+16)/3=38/3=12.66667

$$A_e = 1 - \frac{1}{12.66667}\left[\frac{1}{3}\left(10^{1-e} + 12^{1-e} + 26^{1-e}\right)\right]^{1/1-e}$$

-When considering the number of components and per-capita incomes:

x_i: 5 4 4

n_i: 2 3 4 N=9

$\bar{x}$=(5·2+4·3+4·4)/9=38/9=4.22223

$$A_e = 1 - \frac{1}{4.22223}\left[\frac{1}{9}\left(5^{1-e}\cdot 2 + 4^{1-e}\cdot 3 + 4^{1-e}\cdot 4\right)\right]^{1/1-e}$$

- when considering equivalent components and equivalent incomes:

x_i:	6.3735	5.8766	6.4988	
n_i:	1.569	2.042	2.462	N=6.073

$\overline{x}$=(6.3735·1.569+5.8766·2.042+6.4988·2.462)/6.073=38/6.073=6.2572

$$A_e = 1 - \frac{1}{6.2572}\left[\frac{1}{6.073}\left(6.3735^{1-e}\cdot 1.569 + 5.8766^{1-e}\cdot 2.042 + 6.4988^{1-e}\cdot 2.462\right)\right]^{\frac{1}{1-e}}$$