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# Preface

This course is dedicated to the students of Executive Business Administration (Faculty of Management, Computer Science and Finance, Wrocław University of Economics); its content is prepared in close relationship with the syllabus of Mathematical Statistics, which is obligatory, encompassing 16 hours of lectures and 14 hours of classes.

Mathematical statistics is a very rich field of science. For the purpose of this course, only elementary concepts were chosen so that they could be presented within the limited number of hours planned for the syllabus. First chapters introduce the basic notions of mathematical statistics – probability, random variable, distribution – without which more advanced concepts could not be presented. In what follows, some most important distributions, both discrete and continuous, are presented. The third, most practical part, introduces concepts of statistical inferring. The procedures described in these last chapters have a great impact on real life and decision-making. Testing hypotheses is a basis for introducing innovations on the market (e.g., new drugs) and making decisions on investments. The final chapter contains a set of exercises for further practice. Each exercise is provided with an answer (often with a minor hint) and, as a major hint, with a reference to a relevant chapter which discusses a given problem.

The aim of this course is to present main ideas in a very simple way, illustrated with numerous problems, solved step by step, and to show some means of capturing the uncertainty that is unavoidably present in every aspect of human life.